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Corrigendum

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The authors have discovered that they made numerous errors in their paper. Details are as follows:

1. In equation (3), the factor $-\mathbf{G}+\omega$ should be omitted and γ should be replaced by λ.

2. In the line following equation (3), $[(\lambda+\omega^2)-G]^1$ should be replaced by $[(\lambda+\omega^2)-G]^{-1}$.

3. The number '2' should be omitted from the line following equation (12), the second line of equation (13), the first, third, and last lines of equation (16), the second line of equation (21), and the third, fourth, and ninth terms in equation (29).

4. In general, the last two terms in equation (16) need not vanish on $\partial B_\rho(\mathbf{x})$ in the limit $\rho \to 0$. The terms should be left as: $\mathbf{T}_1 = c_1 \lim_{\rho\to 0} \int_{\partial B_\rho} \psi_{-\nu}(\mathbf{y}) d\Omega$ for the first term and $\mathbf{T}_2 = \mathrm{i}c_1 \lim_{\rho\to 0} \int_{\partial B_\rho} (\mathbf{a}\cdot\nu)\psi(\mathbf{y})\mathrm{d}\Omega$ for the second term.

5. In equation (17), the factor 'u^2' should be replaced by 'μ^2'.

6. Add the following term to equation (21): $\mathbf{T}_1 + \mathbf{T}_2 - 2c_1 \lim_{\rho\to 0} \int_{\partial B_\rho} \mu^{-1}(\nu\cdot\nabla\mu)\psi(\mathbf{y})\mathrm{d}\Omega$.

7. Add $-K_2(u)(\nabla u/u)^2$ to the first equation of the second line following equation (22).

8. Replace $2\int_{\mathbb{R}^3} w(\nabla\mu\cdot\mathbf{u}-\mu^2)[K_2(u)/u^2]\psi\mathrm{d}\mathbf{y}$ in equation (25) by

$$-\int_{\mathbb{R}^3} w[K_2(u)/u][\nabla\mu/\mu]^2 u - 4[\mathbf{u}\cdot\nabla\mu/u] + 3[\mu^2/u]\psi\mathrm{d}\mathbf{y}.$$

9. Add the term $\mathrm{i}w\mathbf{z}\times(\nabla\times\bar{\mathbf{a}})$ to the second equation for ∇w in (27) and replace $\mathrm{i}w(\mathbf{z}\cdot\nabla)\cdot\bar{\mathbf{a}}$ by $\mathrm{i}w(\mathbf{z}\cdot\nabla)\bar{\mathbf{a}}$ (i.e., without the dot after the ∇).

10. In the expression for $[\nabla w\cdot\nabla\mu/\mu]$ in equation (28), the term $\mathrm{i}w[\nabla\mu/\mu]\cdot(\mathbf{z}\times(\nabla\times\bar{\mathbf{a}}))$ should be added.

11. In equation (29), the factor μ^2 should be removed from outside the parentheses. Replace $\bar{\mathbf{a}}$ by $\mathbf{a}$ in the third term. In the second from the last term, replace the quantity in brackets by:

$$\|\mathbf{z}\|\Delta\mu + 3[\mu/\|\mathbf{z}\|] - 4[\mathbf{z}\cdot\nabla u/\|\mathbf{z}\|] + (\nabla\mu)^2[\|\mathbf{z}\|/\mu].$$

Finally, add the following term to that equation:

$$[\hbar^2 c/2\pi^2]\beta c_1 \lim_{\rho\to 0}\int_{\partial B_\rho} [\mathrm{i}\mathbf{a}\cdot\nu - 2(\nu\cdot\nabla\mu/\mu)]\psi(\mathbf{y}) + \psi_{-\nu}(\mathbf{y})\mathrm{d}\Omega.$$

12. Replace equation (30) by

$$S(\psi)(x) = \beta[\hbar^2 c/2\pi^2][-\mu^2]\int_{\mathbb{R}^3}\left\{\|\mathbf{x}-\mathbf{y}\|^{-1} - 4\pi\delta(\mathbf{x}-\mathbf{y})\right\}$$
$$\times\{\mathbf{K}_2[\mu\|\mathbf{x}-\mathbf{y}\|]/\|\mathbf{x}-\mathbf{y}\|\}\,\psi(\mathbf{y})\mathrm{d}\mathbf{y} + \mathbf{T}_1.$$

There are then obvious changes in equation (31).

13. Replace 2π by 4π in equation (37), and add $\beta[\hbar^2 c/2\pi^2][\mathbf{T}_1 + \mathbf{T}_2]$.

14. Add $\beta[\hbar^2 c/2\pi^2]\mathbf{T}_2$ to equations (38*a*) and (38*b*). In addition, by using the results in the discussion in the first paragraph of section 5, equation (38) can be expressed in the form:

$$\begin{aligned} S(\psi)(\mathbf{x}) = \frac{\hbar^2 c}{2\pi^2}\beta\int_{\mathbb{R}^3} \mathrm{e}^{\mathrm{i}\bar{a}\cdot(\mathbf{x}-\mathbf{y})}\Bigg\{ & \|\mathbf{a}(\mathbf{y})\|^2\frac{\mu K_1(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|} \\ & -\frac{\mu^2 K_2(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|}[\|\mathbf{x}-\mathbf{y}\|^{-1}(1+\mathrm{i}\mathbf{a}\cdot\mathbf{x}) - 4\pi\delta(\mathbf{x}-\mathbf{y})] \\ & -\frac{e\lambda\mu}{\hbar c}\frac{K_1(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|^2\|\mathbf{x}+\mathbf{y}\|}(\mathbf{a}\cdot\mathbf{x})(\mathbf{x}\cdot\mathbf{y}) \\ & +\frac{\mathrm{i}e\lambda\mu^2}{\hbar c}\frac{K_2(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|\,\|\mathbf{x}+\mathbf{y}\|}(\mathbf{x}\cdot\mathbf{y} + \|\mathbf{y}\|^2) \\ & +\frac{\mu}{4}\left(\frac{e\lambda}{\hbar c}\right)^2\frac{K_1(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|}\|\mathbf{y}\|^2 \\ & +\frac{\mathrm{i}e\lambda\mu}{2\hbar c}\frac{K_1(\mu\|\mathbf{x}-\mathbf{y}\|)}{\|\mathbf{x}-\mathbf{y}\|^2\|\mathbf{x}+\mathbf{y}\|}(\|\mathbf{x}\|^2 - \|\mathbf{y}\|^2)\Bigg\}\,\psi(\mathbf{y})\mathrm{d}\mathbf{y} + \mathbf{T}_1 + \mathbf{T}_2. \end{aligned}$$

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